MPRI – Course on Concurrency

Probabilistic methods in Concurrency

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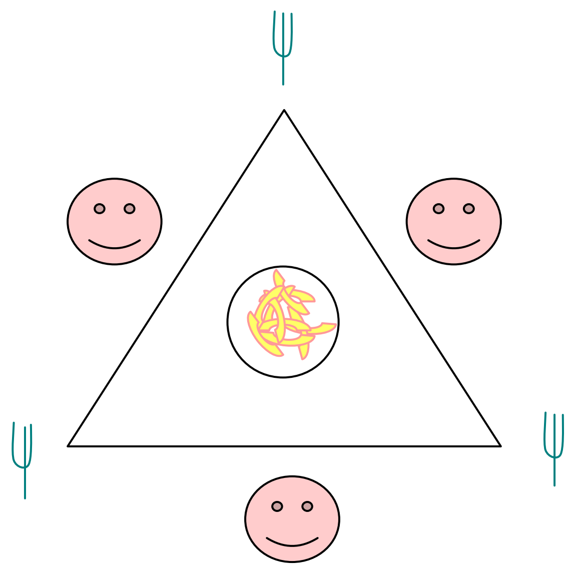
Page of the course:
http://pauillac.inria.fr/~leifer/teaching/mpri-concurrency-2004/

Filling the gap between the pi-calculus and the asynchronous pi-calculus

- The pi-calculus is strictly more expressive than the asynchronous pi-calculus
- On the other hand, the asynchronous pi-calculus can be implemented in a distributed way, while the (synchronous) pi-calculus cannot
- It is possible to enrich the asynchronous pi-calculus with a probabilistic (internal) choice so to achieve the same expressive power as the pi-calculus
- The idea is to implement the mixed choice construct via a randomized algorithm which is based on (an extension of) the randomized solution of Lehmann and Rabin to the dining philosophers

The dining philosophers

- Each philosopher needs exactly two forks
- Each fork is shared by exactly two philosophers
- A philosopher can access only one fork at the time



Intended properties of solution

- **Deadlock freedom** (aka **progress**): if there is a hungry philosopher, a philosopher will eventually eat
- **Starvation freedom**: every hungry philosopher will eventually eat (but we won't consider this property here)
- **Robustness wrt a large class of adversaries**: *Adversaries decide who does the next move (schedulers)*
- **Fully distributed**: no centralized control or memory
- **Symmetric**:
  - All philosophers run the same code and are in the same initial state
  - The same holds for the forks

Non-existence of a "deterministic" solution

- Lehman and Rabin have shown that there does not exist a "deterministic" (i.e. non-probabilistic) solution to the dining philosophers, satisfying all properties listed in previous slide.

- The proof proceeds by proving that for every possible program we can define an adversary (scheduler) which preserves the initial symmetry

- **Note:** Francez and Rodeh did propose a "deterministic" solution using CSP, similar to the solution in pi-calculus given in Lecture 6. The solution to this apparent contradiction is that CSP and the pi-calculus cannot be implemented in a fully distributed way because of the (mixed) guarded choice construct

The algorithm of Lehmann and Rabin

1. Think
2. randomly choose fork in {left,right}   %commit
3. if taken(fork) then goto 3
   else take(fork)

4. if taken(other(fork)) then {release(fork); goto 2}
   else take(other(fork))
5. eat
6. release(other(fork))
7. release(fork)
8. goto 1

Correctness of the algorithm of Lehmann and Rabin

- **Theorem:** for every **fair** adversary, if a philosopher becomes hungry, then a philosopher (not necessarily the same) will eventually eat with probability 1.

- **Proof:** the original proof is not fully formalized it is difficult to follow. There is a proof by Segala and Lynch, using **Progress Statements**, which is easier

- **Question:** why the fairness requirement? Can we write a variant of the algorithm which does not require fairness?

Progress statements

- Progress statements
  - Proposed by Lynch and Segala
  - *A* formal method to analyze probabilistic algorithms

- **Definition (progress statements)**
  - Given sets of states S, T, and a class of adversaries *A*, we write

    S –*A*,p-> T

    if, under any adversary in *A*, from any state in S, we eventually reach a state in T with probability at least p

  - Furthermore, we write

    S unless T

    if, whenever from a state in S we do not reach a state in T, we remain in S (possibly in a different state of S)

History insensitivity

- **Definition:** a class of adversaries A is history-insensitive if: for every $\alpha \in A$, and for every fragment of execution e, there exists $\alpha' \in A$ such that
  for every fragment of execution e', $\alpha'(e') = \alpha(ee')$

- **Proposition:** The class of fair adversaries is history-insensitive

  Proof: Given α and e, define $\alpha'(e') = \alpha(ee')$. Clearly α' is still fair

Progress statements

- **Some useful properties**

  - If A is history-insensitive , S –A,p-> T, and T –A,q-> U, then
    S –A,pq-> U

  - If S_1 –A,p_1-> T_1, and S_2 –A,p_2-> T_2 , then
    $S_1 \cup S_2$–A,p-> $T_1 \cup T_2$
    where $p = \min\{p_1,p_2\}$

  - S –A,1-> S

  - If A is history-insensitive and S –A,p-> T and S unless T, and $p > 0$, then
    S –A,1-> T

Proof of d-f for the dining philosophers

- Proof of deadlock-freedom for the algorithm of Lehmann and Rabin for the Dining Philosophers
  - We will show that under a fair adversary scheduler we have deadlock-freedom (and livelock-freedom), i.e. if a philosopher gets hungry, then with probability 1 some philosopher (not necessarily the same) will eventually eat.

Proof of d-f: the algorithm

State	action	description
• R	think or get hungry	reminder region
• F	flip	ready to toss
• W	wait	waiting for first fork
• S	second	checking second resource
• D	drop	dropping first resource
• P	eat	pre-critical region
• C	exit	critical region
• E_F	dropF	drop first fork
• E_S	dropS	drop second fork
• E_R	rem	move to reminder region

T

Example of verification: The dining philosophers

- Let us introduce the following global (sets of) states

  Try : at least one phil is in T={F,W,S,D,P}

  Eat : at least one phil is in C

  RT : at least one phil is in T, all the others are in T, R or E_R

  Flip : at least one phil is in F

  Pre : at least one phil is in P

  Good : at least one process is in a "good state", i.e. in {W,S} while his second fork f is not the first fork for the neighbor (i.e. the neighbor is not committed to f)

- We want to show that Try -A,1-> Eat for A = fair adv

Example of verification: The dining philosophers

- We can prove that, for the class of fair adversaries A (omitted in the following notation):
  - Try -1-> RT $\cup$ Eat
  - RT -1-> Flip $\cup$ Good $\cup$ Pre
  - Flip -1/2-> Good $\cup$ Pre
  - Good -1/4-> Pre
  - Pre -1-> Eat
- Using the properties of progress statements we derive

  Try -1/8-> Eat

- Since we also have Try unless Eat, we can conclude

Encoding π into π_{pa}

- **[[ ]] :** $\pi \rightarrow \pi_{pa}$
- **Fully distributed**

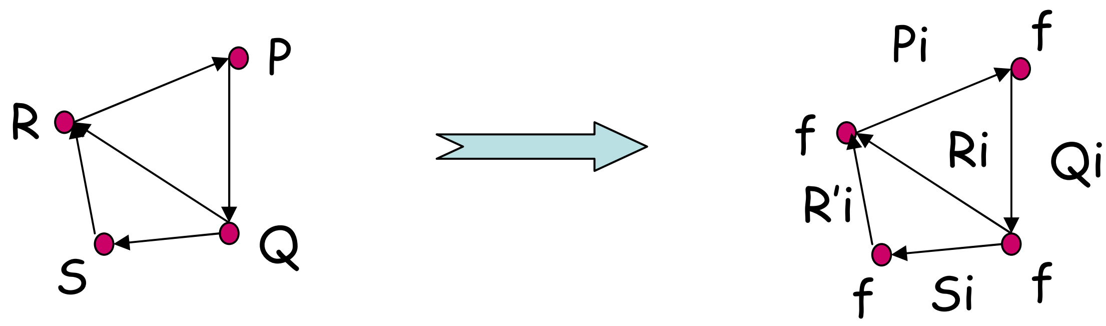
  [[P | Q]] = [[P]] | [[Q]]
- **Uniform**

  [[P σ]] = [[P]] σ
- **Correct wrt a notion of probabilistic testing semantics**

  P must O iff [[P]] must [[O]] with prob 1

Encoding π into π_{pa}

- Idea:
  - Every mixed choice is translated into a parallel comp. of processes corresponding to the branches, plus a lock f
  - The input processes compete for acquiring both its own lock and the lock of the partner
  - The input process which succeeds first, establishes the communication. The other alternatives are discarded



The problem is reduced to a generalized dining philosophers problem where each fork (lock) can be adjacent to more than two philosophers

Problems

- Wrt to our encoding goal, the algorithm of Lehmann and Rabin has two problems:

1. It only works for certain kinds of graphs

2. It works only for **fair** schedulers

- Problem 2 however can be solved by replacing the busy waiting in step 3 with suspension.
  [Duflot, Friburg, Picaronny 2002] – see also [Herescu's PhD thesis]

The algorithm of Lehmann and Rabin

1. Think
2. randomly choose fork in {left,right}   %commit
3. if taken(fork) then goto 3
   else take(fork)

4. if taken(other(fork)) then {release(fork); goto 2}
   else take(other(fork))
5. eat
6. release(other(fork))
7. release(fork)
8. goto 1

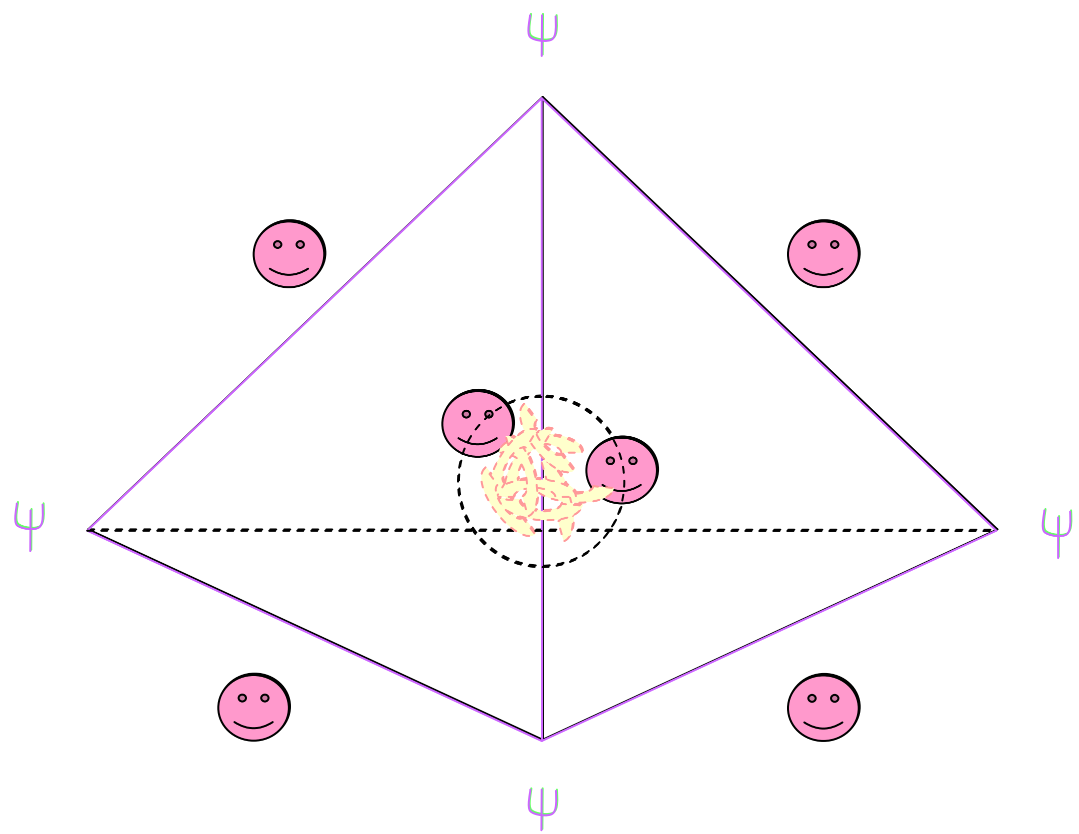
The algorithm of Lehmann and Rabin modified so to eliminate the need of fairness

1. Think
2. randomly choose fork in {left,right}   %commit
3. if taken(fork) then wait;
4. take(fork)

5. if taken(other(fork)) then {release(fork); goto 2}
   else take(other(fork))
6. eat
7. release(other(fork))
8. release(fork)
9. goto 1

Dining Phils: generalized case

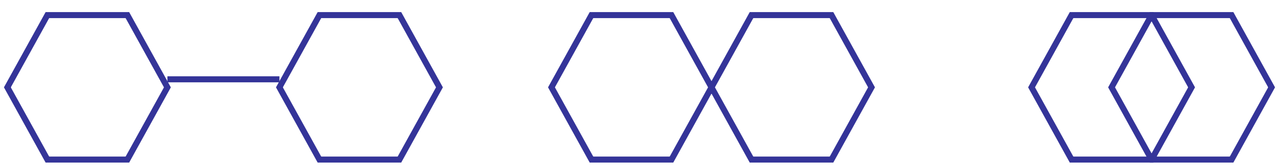
Each fork can be shared by more than two philosophers



Dining Phils: generalized case

- **Theorem:** The algorithm of Lehmann and Rabin is deadlock-free if and only if all cycles are pairwise disconnected

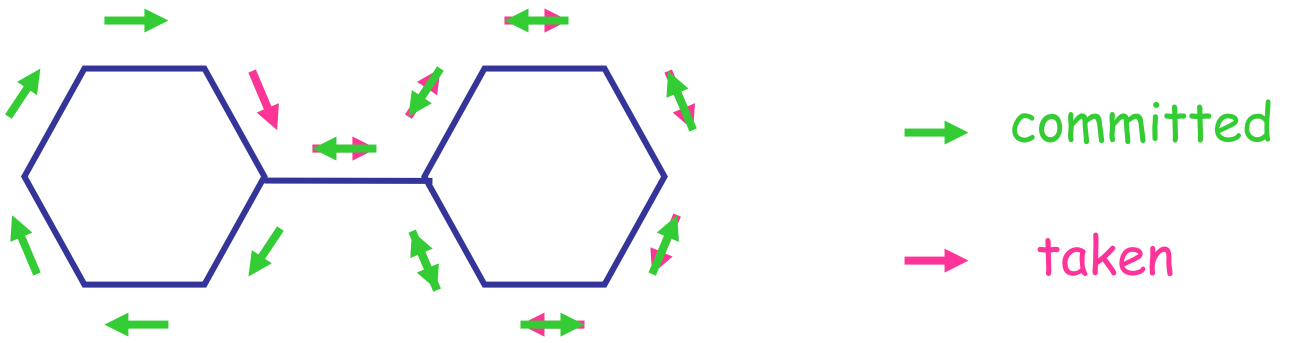
- There are essentially three ways in which two cycles can be connected:



Proof of the theorem

- If part) Each cycle can be considered separately. On each of them the classic algorithm is deadlock-free. Some additional care must be taken for the arcs that are not part of the cycle.

- Only if part) By analysis of the three possible cases. Actually they are all similar. We illustrate the first case



Proof of the theorem

- The initial situation has probability $p > 0$
- The scheduler forces the processes to loop
- Hence the system has a deadlock (livelock) with probability p

- Note that this scheduler is **not fair**. However we can define even a fair scheduler which induces an infinite loop with probability > 0. The idea is to have a scheduler that "gives up" after n attempts when the process keep choosing the "wrong" fork, but that increases (by f) its "stubborness" at every round.

- With a suitable choice of n and f we have that the probability of a loop is $p/4$

Solution for the Generalized DP

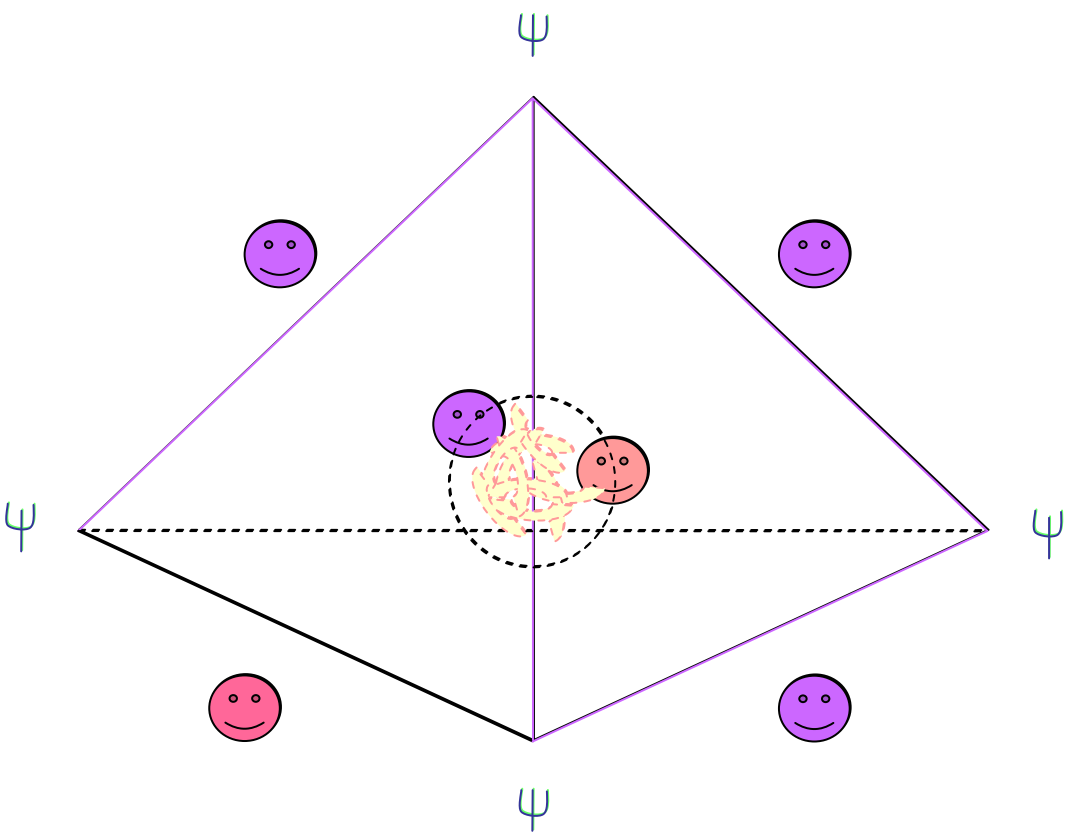
- As we have seen, the algorithm of Lehmann and Rabin does not work on general graphs
- However, it is easy to modify the algorithm so that it works in general
- The idea is to reduce the problem to the pairwise disconnected cycles case:

  Each fork is initially associated with one token. Each phil needs to acquire a token in order to participate to the competition. After this initial phase, the algorithm is the same as the Lehmann & Rabin'

- **Theorem:** The competing phils determine a graph in which all cycles are pairwise disconnected

  Proof: By case analysis. To have a situation with two connected cycles we would need a node with two tokens.

Dining Phils: generalized case



Reduction to the classic case: each fork is initially associated with a token. Each phil needs to acquire a token in order to participate to the competition. The competing phils determine a set of subgraphs in which each subgraph contains at most one cycle